Gary Becker’s Model of Time Allocation and Consumer Choice

In Becker’s reformulation of the household choice model, each household still acts as if it were an individual (this is called unified preferences because it ignores intra-household allocation issues). Another way to think about this is that each household has just one person.

This individual maximizes her utility that is a function of various activities she likes to do. Some examples are sleeping, eating, watching TV, volunteering, going to parties, playing soccer and perhaps even working. Some misleadingly, Becker calls these activities (basic) commodities. I prefer to think of these as activities. For those of you who are familiar with Amartya Sen’s capabilities framework, the Z’s can be thought of as functionings, which Sen defines as ”things we like to do or states we like to be”. For example, eating is something we like to do, being healthy is a state will like to be.

Suppose there are two activities, $Z_1$ and $Z_2$ that this person likes. Her utility function is,

$$U = U(Z_1, Z_2)$$ (1)

We will assume as usual that $U_1 > 0$, $U_2 > 0$, $U_{11} < 0$, $U_{22} > 0$. These conditions ensure that utility is an increasing and concave function of the two activities (i.e. positive but diminishing marginal utility).

Each activity is ”produced” by combining market purchased goods and time (In Gronau’s model, some goods can be produced at home as well). For example, eating an activity that is ”produced” or ”carried out” by combining market purchased food and time spent on eating the food. The activity of education is achieved by combining
goods and services with study time.

The activity production functions are,

\[ Z_1 = Z_1(X_1, t_1) \]  \hspace{1cm} (2) \\
\[ Z_2 = Z_2(X_2, t_2) \]  \hspace{1cm} (3)

In order to purchase the goods needed for consumption, this person works in the labor market. Her budget constraint is,

\[ P_1 X_1 + P_2 X_2 = wL + V \]  \hspace{1cm} (4)

where \( P_1 \) and \( P_2 \) are the prices of consumption goods, \( w \) is the market wage, \( L \) is the time spent working, and \( V \) is the nonearned income.

We can safely assume that the budget constraint is binding (holds with equality) because the utility function is monotonically increasing in both activities.

Her time constraint is,

\[ t_1 + t_2 + L = T \]  \hspace{1cm} (5)

where \( T \) is her time endowment.

We can combine the time constraint with the budget constraint to obtain the full income constraint.

\[ P_1 X_1 + P_2 X_2 + wt_1 + wt_2 = wT + V \]  \hspace{1cm} (6)

We can rearrange terms, can write this as

\[ P_1 X_1 + wt_1 + P_2 X_2 + wt_2 = wT + V \]  \hspace{1cm} (7)

The first two terms in the LHS is the total expenditure on activity \( Z_1 \), third and fourth terms are the total expenditure on activity \( Z_2 \). Note that the expenditure on each activity has two components; the first is the money cost of the goods used for this activity (e.g. the ball used for soccer), the second is the opportunity cost of the time used for this activity (lost earnings due to the time spent playing soccer). The right hand side of the constraint represents the "full income", or the maximum possible income of this individual. This is what she would earn if she allocates her entire time endowment to work.
1 Solution when Goods and Time are Perfect Complements in each Activity

Becker’s simple model makes a further assumption that goods and time are perfect substitutes in the production of each activity. This means that goods and time must be always used in fixed proportions to produce this activity. For example, in order to eat a pizza, you always need one pizza and 10 minutes. In order to make the pizza, you always need one pound of dough, one pound of cheese and one hour. This allows him to write the activity production functions in the following form,

\[ X_i = b_iZ_i \]
\[ t_i = c_iZ_i \]

where \( b_i \) and \( c_i \) are constant parameters that represent the fixed input intensity of each activity. If \( b_1 \) is 2, for example, each unit of activity \( Z_1 \) is produced with 2 units of the good \( X_1 \).

This assumption allows him to write the full income budget constraint in the following simpler form,

\[
(P_1b_1 + wc_1)Z_1 + (P_2b_2 + wc_2)Z_2 = wT + V \tag{8}
\]

Note that this budget constraint has now been expressed as a function of the two activities, rather than the four inputs. The coefficient of each activity is the unit cost of that activity. For example, in order to produce one unit of \( Z_1 \), she has to spend \( P_1b_1 + wc_1 \), where \( P_1 \) is the price of one good, and \( b_1 \) is how many goods she needs to produce one unit of \( Z_1 \). Similarly, \( w \) is the price of one unit of time, and \( c_1 \) is how much time she needs to produce one unit of \( Z_1 \). Because these input intensities are fixed, her maximization problem can be reduced to one that has only two choice variables, \( Z_1 \) and \( Z_2 \). In other words, all she needs to do is to choose the optimal level of each activity, because once that is known, the fixed amount of goods and time needed to carry out this activities are automatically determined. In a more complex model where goods and time can be substituted in the production of activities (i.e.
there are no fixed proportions of time and goods), she will have to choose all four inputs individually. Such a model is outlined in the next section.

But for now, we can write her optimization problem as the following Lagrangian,

$$\max_{Z_1, Z_2} U = U(Z_1, Z_2) + \lambda(wT + V - \pi_1 Z_1 - \pi_2 Z_2)$$  \hspace{1cm} (9)

Here,

$$\pi_1 = P_1 b_1 + w c_1$$
$$\pi_2 = P_2 b_2 + w c_2$$

represent the combined time and goods cost of each activity.

The first-order conditions of this problem take the following familiar form,

$$\frac{\partial U}{\partial Z_1} = \lambda \pi_1$$
$$\frac{\partial U}{\partial Z_2} = \lambda \pi_2$$

$$\pi_1 Z_1 + \pi_2 Z_2 = wT + V$$

This is very similar to the FOC’s we would have obtained with a simple consumption-leisure time model. The relative prices in this model, however, are more realistic and have richer implications.

By combining the first two FOC’s, we see that the optimum \((Z_1^*, Z_2^*)\) occurs at a point where the indifference curve between \(Z_1\) and \(Z_2\) and the budget constraint have the same slope.

$$\frac{\partial U}{\partial Z_1} = \frac{\pi_1}{\pi_2} = \frac{P_1 b_1 + w c_1}{P_2 b_2 + w c_2}$$  \hspace{1cm} (10)

The third FOC tells us the optimum must lie on the budget constraint. Taking these two conditions together, we see that the optimum lies at the tangency point between the indifference curve and budget constraint.

Note that the slope of the budget constraint, or the relative price of the activities, can be influenced by changes in seven different parameters. Three of these are prices,
and the other four are the input intensities of the activities. Changes in any of these can cause a substitution effect. This is perhaps the most important contribution of Becker’s time allocation model. To see this more clearly, let’s examine how each of these parameters influences the relative price \( R = \frac{\pi_1}{\pi_2} \). The following effects are obvious,

\[
\frac{\partial R}{\partial P_1} > 0, \frac{\partial R}{\partial P_2} < 0, \frac{\partial R}{\partial b_1} > 0, \frac{\partial R}{\partial b_2} < 0, \frac{\partial R}{\partial c_1} > 0, \frac{\partial R}{\partial c_2} < 0
\] (11)

This tells us that the following events would make us substitute away from activity \( Z_1 \) towards activity \( Z_2 \).

1. Increase in the price of goods used for activity \( Z_1 \)
2. Decrease in the price of goods used for activity \( Z_2 \)
3. Increase in the goods and/or time intensity of activity \( Z_1 \)
4. Decrease in the goods and/or time intensity of activity \( Z_2 \)

When the price of soccer balls decreases, she will substitute her time away from watching TV towards playing soccer. If the price of both soccer balls and DVDs fall by the same proportion, the relative price of watching a DVD could decrease if the activity of watching movies is more goods intensive than the activity of playing soccer. In other words, the impact of a price change of a good is greater on activities that are more goods intensive.

The time and goods intensities of activities may change with technological improvements in household consumption activity. A microwave for example will reduce the time intensity of cooking and eating. Pipe-borne hot water will reduce the time intensity of showering. Netflix and HBO will reduce the time intensity of watching movies. A more efficient heating system will reduce the goods intensity of home heating. The availability of public transport will reduce the goods intensity of traveling to work.
The impact of an increase in wages on the relative price of the activities is trickier to establish because the wage is the implicit price of the time used for both activities. However, it is reasonable to suppose that the impact of a wage increase would be to increase the relative price of the more time-intensive activity. To see this, let’s take the partial derivative of the relative price with respect to the wage.

\[ \frac{\partial R}{\partial w} = \frac{(P_2 b_2 + wc_2) c_1 - (P_1 b_1 + wc_1) c_2}{(P_2 b_2 + wc_2)^2} = \frac{P_2 b_2 c_1 - P_1 b_1 c_2}{(P_2 b_2 + wc_2)^2} \] (12)

The denominator is always positive. The numerator is positive, and therefore, a wage increase will increase the relative price of \( Z_1 \) if,

\[ P_2 b_2 c_1 - P_1 b_1 c_2 > 0 \] (13)

This leads to the following condition for a positive effect of wage on the relative price of \( Z_1 \)

\[ \frac{c_1}{c_2} > \frac{P_1 b_1}{P_2 b_2} \] (14)

If the activity \( Z_1 \) is sufficiently more time-intensive than activity \( Z_2 \), an increase in wage will lead her to substitute from \( Z_1 \) to \( Z_2 \). The lower the relative price of goods used in the activity \( Z_1 \) and lower the relative goods intensity of \( Z_1 \), this substitution will happen at a lower level of \( c_1 \), i.e. the inequality condition will be satisfied at a lower level of \( \frac{c_1}{c_2} \).

2 Closed Form Solutions

In order to get explicit solutions for the demand functions, we need to make assumptions about the utility function. Suppose the utility function takes the familiar Cobb-Douglas form.

\[ U(Z_1, Z_2) = Z_1^\alpha Z_2^\beta \] (15)
The corresponding marginal utilities are,
\[
\frac{\partial U}{\partial Z_1} = \alpha Z_1^{\alpha-1} Z_2^\beta \\
\frac{\partial U}{\partial Z_2} = \beta Z_1^\alpha Z_2^{\beta-1}
\]
and the MRS is
\[
\frac{\partial U}{\partial Z_1} \bigg/ \frac{\partial U}{\partial Z_2} = \frac{\alpha Z_2}{\beta Z_1} \tag{16}
\]
Using this in the FOC’s, we get,
\[
\alpha \pi_1 Z_1 = \beta \pi_2 Z_2 \tag{17}
\]
Plugging this in the constraint, we get
\[
\pi_1 Z_1 + \frac{\alpha}{\beta} \pi_1 Z_1 = wT + V \tag{18}
\]
Solving for \(Z_1\) etc, we get explicit closed-form solutions for each activity,
\[
Z_1^* = \frac{\alpha}{\alpha + \beta} \frac{wT + V}{\pi_1} \\
Z_2^* = \frac{\beta}{\alpha + \beta} \frac{wT + V}{\pi_2}
\]
Because of the assumption of fixed time intensities, we can easily obtain the demand functions for goods and time.
\[
X_1^* = \frac{\alpha}{\alpha + \beta} \frac{b_1 (wT + V)}{P_1 b_1 + wc_1} \\
X_2^* = \frac{\beta}{\alpha + \beta} \frac{b_2 (wT + V)}{P_2 b_2 + wc_2} \\
t_1^* = \frac{\alpha}{\alpha + \beta} \frac{c_1 (wT + V)}{P_1 b_1 + wc_1} \\
t_2^* = \frac{\beta}{\alpha + \beta} \frac{c_2 (wT + V)}{P_2 b_2 + wc_2}
\]
Using these demand functions, we can perform comparative static simulations to see how this individual’s behavior responds to changes in exogenous variables. To do this formally, we need to take the derivatives of the demand functions with respect to each parameter.
1. When our preference for the first activity $\alpha$ increases, our demand for $X_1$ and $t_1$ goes up, and the demand for $X_2$ and $t_2$ goes down.

2. Similarly, when our preference for the second activity $\beta$ increases, our demand for $X_2$ and $t_2$ goes up, and the demand for $X_1$ and $t_1$ goes down.

3. When an activity becomes more goods intensive, our demand for the goods used in that activity increases, and the demand for time used in that activity decreases. This has no effect on the other activity.

4. When either the time endowment $T$ or the nonearned income $V$ increases, the demand for all four inputs increases.

5. When the price of the good used for an activity goes up, the demand for both the good and the time used for that activity goes down, i.e. there is a substitution away from that activity. There is no effect on the other activity (because the Cobb-Douglas utility function has the odd property of holding the expenditure shares constant).

Wage changes induce both income and substitution effects that go in opposite directions. The net effect of a wage change on the demand depends on other parameters. In order to find these, we need to take the derivative of the demand functions with respect to the wage.

Question: Take these derivatives and find the conditions under which the income and substitution effects dominate.

3 A More General Formulation of the Activity Production Functions

In general, the problem can be stated as the following Lagrangian;

$$\max_{X_1, X_2, t_1, t_2} U = U(Z_1(X_1, t_1), Z_2(X_2, t_2)) + \lambda(wT + V - P_1 X_1 - wt_1 - P_2 X_2 - wt_2) \quad (19)$$
The first order conditions with respect to the four choice variables and \( \lambda \) are,

\[
\frac{\partial U}{\partial Z_1} \frac{\partial Z_1}{\partial X_1} = \lambda P_1 \\
\frac{\partial U}{\partial Z_2} \frac{\partial Z_2}{\partial X_2} = \lambda P_2 \\
\frac{\partial U}{\partial Z_1} \frac{\partial Z_1}{\partial t_1} = \lambda w \\
\frac{\partial U}{\partial Z_2} \frac{\partial Z_2}{\partial t_2} = \lambda w 
\]

\[P_1X_1 + P_2X_2 + wt_1 + wt_2 = wT + V\]

By solving these five equations, we can find the demand functions for goods and time.

\[X_1^* = X_1^*(P_1, P_2, w, T, V)\]
\[X_2^* = X_2^*(P_1, P_2, w, T, V)\]
\[t_1^* = t_1^*(P_1, P_2, w, T, V)\]
\[t_2^* = t_2^*(P_1, P_2, w, T, V)\]

Once we know the demand functions, we can perform comparative static exercises by determining the impact of changing the exogenous variables on the demand for each good and each time.

The methodology of this analysis is the same as in the previous example, and the results are also by and large the same. There is one significant insight that is obtained from this generalized model that was not present in the simple model. To see this, examine the following condition that is obtained by manipulating the FOCs.

\[
\frac{\partial U}{\partial t_i} \frac{\partial X_i}{\partial z_i} = \frac{\partial X_i}{\partial t_i} |_{z_i} = \frac{w}{P_i} \tag{20}
\]

Here, \( \frac{\partial X_i}{\partial t_i} |_{z_i} \) represents the (marginal) technical rate of substitution between goods and time in the production of the activity \( Z_i \). The TRS is the slope of the iso-quant. This condition is analogous to the familiar optimal condition in producer theory that
the TRS must equal the relative factor price \( w/P_i \). Therefore, when the real wage increases, with a compensating income change to hold the level of \( Z_i \) constant, we should expect a substitution away from time towards goods in each activity. As relative factor prices change, we will now see two behavioral changes: 1) there will be an increase (a decrease) in activities that are intensive in the relatively cheaper (expensive) factor, 2) there will be an increase (decrease) in the intensity of the relatively cheaper (expensive) factor in both activities. The first effect is present in both the simple and general models, but the second effect is present only in the general model. The reason for this is simple: In the simpler model, we assumed that time and goods are perfect complements in the production of each activity. Therefore, the iso-quants have the familiar right-angled shape that we have seen with perfect complements, and the optimum factor choice is not sensitive to relative price changes. However, the utility function allowed the individual to substitute away from the relatively more expensive activity towards the cheaper one. In the general model, substitution is possible between and within activities.